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# From computer systems to power systems: using stochastic network calculus for flexibility analysis in power systems

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# Abstract

As power systems transition from controllable fossil fuel plants to variable renewable sources, managing power supply and demand fluctuations becomes increasingly important. Novel approaches are required to balance these fluctuations. The problem of determining the optimal deployment of flexibility options, considering factors such as timing and location, shares similarities with scheduling problems encountered in computer networks. In both cases, the objective is to coordinate various distributed units and manage the flow of either data or power. Among the methods for scheduling and resource allocation in computer networks, stochastic network calculus (SNC) is a promising approach that estimates worst-case guarantees for Quality of Service (QoS) indicators of computer networks, such as delay and backlog. Promising QoS indicators in the power system are given by the amount of stored energy, the serviced demand, and the demand elasticity. In this work, we investigate SNC for its capabilities and limitations to quantify flexibility service guarantees in power systems. We generate and aggregate stochastic envelopes for random processes, which was found useful for modeling flexibility in power systems at multiple time scales. In a case study on the reliability of a solar-powered car charging station, we obtain similar results as from a mixed-integer linear programming problem, which provides confidence that the chosen SNC approach is suitable for modeling power system flexibility.

**Keywords:** Power system flexibility, Stochastic network calculus, Quality of Service indicators, Network engineering, Flexibility service guarantees

# Introduction

The continuous availability of electricity is crucial for modern societies, making power systems highly demanding in terms of reliability and supply (Jiang et al. 2016). With the increasing integration of variable renewable energy sources (vRES) and the need to reduce greenhouse gas emissions, guaranteeing the end-users energy demand to be covered at all times and locations becomes more and more challenging. This



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necessitates the development of novel approaches to capture the evolving dynamics of power systems and quantify their flexibility.

The literature shows a growing interest in identifying generic indicators to describe power system flexibility (Papaefthymiou et al. 2018; Bhuiyan et al. 2022), which then can be used to evaluate the system's ability to provide power to end-users reliably. This concept is closely related to Quality of Service (QoS) indicators in computer networking, which describe measures for the overall performance of a service provided to network end-users, including factors such as delay and data backlog. Moreover, the future energy system and computer networks face the same challenge of sharing multiple distributed resources among nodes while adhering to constraints such as limited transmission or storage capacity without violating their QoS.

Unlike in power systems, methods to estimate worst-case guarantees for the QoS indicators are well known in computer networking. In particular, the mathematical framework of stochastic network calculus (SNC) emerges as a promising modeling tool. SNC bears the great advantage of directly estimating worst-case guarantees for the QoS indicators of computer networks through probabilistic envelopes in the interval domain (Ciucu and Schmitt 2012; Jiang and Liu 2008; Fidler and Rizk 2014). Its envelope-based modeling approach is well-suited for analyzing power systems with a high share of vRES generation (Jiang et al. 2016; Wang et al. 2012), as it can handle a wide range of stochastic processes.

This work investigates the capabilities and limitations of SNC to quantify guarantees for the flexibility of a power system using a set of generic QoS indicators. The generic QoS indicators used within this work are the amount of stored energy in the ESS, the serviced demand, and the demand elasticity, e.g., the time the demand can be delayed. Extending the work of Wang et al. (2012), Raeis et al. (2017), Ghiassi-Farrokhfal et al. (2014), SNC is used to derive worst-case guarantees for the aforementioned QoS indicators. The obtained flexibility service guarantees describe the minimum available flexibility potential of the power system, as well as its maximum flexibility potential required to meet a certain QoS level under a worst-case assumption. These guarantees are referred to as: (i) guaranteed state of charge probability (GSCP), (ii) guaranteed power service probability (GPSP), and (iii) guaranteed temporal service probability (GTSP).

The contributions of this work are threefold:

- 1. It extends the work of how scheduling concepts from computer networking, particularly SNC, can be applied to power system modeling to the dimension of flexibility analysis. The focus is on the capabilities and limitations of quantifying power system flexibility using stochastic envelopes in the interval domain.
- 2. It provides a method for quantifying guarantees for power system flexibility using computer networking concepts. Therefore generic QoS indicators for the available flexibility potential and the flexibility requirements of the power system are used.
- 3. It validates the flexibility service guarantees by comparing it with results obtained from a mixed-integer linear programming (MILP) problem for a case study on the reliability of a solar-powered car charging station. The analytic flexibility service guarantees obtained from SNC align well with the MILP results.

The remainder of this paper is organized as follows: section Related work reviews the relevant literature; section Methodology: SNC concepts for flexibility modeling introduces SNC concepts and their applicability for modeling power system flexibility; section Case study: reliability of solar-powered car charging presents a case study on the feasibility of hundred percent renewable workplace car charging at a small research institute; and section Discussion highlights the capabilities and limitations of SNC concepts to quantify flexibility service guarantees for power systems. Finally, the work is summarized, and future research directions are suggested in section Conclusion.

#### **Related work**

The following presents related work on the general concepts of SNC and its application in power system modeling.

# **Network Calculus concepts**

Network Calculus (NC) is a theory for service guarantee analysis in computer networks. It uses an alternate algebra, the (*min*, +)-algebra, and a unique envelope concept to derive worst-case performance bounds on specific QoS indicators, such as backlog and delay. In order to provide such an analysis, a model of both the flows and the network is required. Modeling the data flows and the service within the framework is done by using cumulative envelope functions from the set of non-negative, non-decreasing functions  $F = \{f(\cdot) : \forall 0 \le a \le b, 0 \le f(a) \le f(b)\}$ , the so-called arrival and service curves. They follow the convention of  $f(x) = 0 \forall x < 0$ .

Arrival curves  $\alpha_A(x)$  describe an upper envelope on the incoming traffic to a system in any interval of time, while service curves  $\beta_S(x)$  describe a lower envelope on the system's service. From these envelopes, worst-case bounds on the backlog, e.g., the amount of data held inside the system and the delay the incoming data experiences, can be analytically calculated. This calculation is usually done by computing the maximum vertical and horizontal deviations between the arrival and the service curves (Ciucu and Schmitt 2012; Le Boudec and Thiran 2001). By exploiting the properties of the (*min*, +)-algebra, where addition becomes the calculation of the infimum and multiplication becomes an addition (Le Boudec and Thiran 2001), complex non-linear computer networks can be transformed into analytically tractable linear systems.

SNC extends the envelope concept of NC by additionally introducing so-called bounding functions  $\epsilon_A$  and  $\epsilon_S$ , which allow the arrival and service curves to be violated with a certain probability P (Ciucu and Schmitt 2012). The bounding functions are defined on the set of non-negative, non-increasing functions  $\overline{F} = f(\cdot) : \forall 0 \le a \le b, 0 \le f(b) \le f(a)$ with the convention of  $f(x) = 1 \forall x < 0$ . The most widely used envelope model of SNC is called statistical sample path envelope and is defined as follows (Jiang and Liu 2008).

**Definition 1** [*Sample-Path-Envelope*] A flow Q has a stochastic upper envelope  $\alpha_Q \in \mathcal{F}$  with bounding function  $\epsilon_Q^u \in \overline{\mathcal{F}}$ , denoted by  $Q \sim \langle \epsilon_Q^u, \alpha_Q \rangle$ , if for all  $0 \le s \le t$  and all  $x \ge 0$ :

$$P\left\{\sup_{0\le s\le t}\left\{\left(\left[Q(t)-Q(s)\right]-\alpha_Q(t-s)\right\}>x\right\}\le \epsilon_Q^u(x)\tag{1}$$

and it has a stochastic lower envelope  $\beta_Q \in \mathcal{F}$  with bounding function  $\epsilon_Q^l \in \overline{\mathcal{F}}$ , denoted by  $Q \sim \langle \epsilon_Q^l, \beta_Q \rangle$ , if for all  $0 \leq s \leq t$  and all  $x \geq 0$  there holds:

$$P\left\{\sup_{0\le s\le t} \{\beta_Q(t-s) - [Q(t) - Q(s)]\} > x\right\} \le \epsilon_Q^l(x)$$
(2)

The strength of the envelope concept is that already a combination of simple functions adequately characterizes a large class of stochastic processes sufficiently well (Fidler and Rizk 2014). One example is the combination of a constant rate sample path envelope  $A(t) = r \cdot t$  and an exponential decaying bounding function  $\epsilon(x) = p \cdot \exp\{-\kappa \cdot x\}$ , in literature generally referred to as the Exponentially Bounded Burstiness EBB model (Mao and Panwar 2006). A full description of NC and SNC, its mathematical derivations, and further applications in the field of computer networking are given by Ciucu and Schmitt (2012), Jiang and Liu (2008), Le Boudec and Thiran (2001).

## Network Calculus in power system modeling

In the first application of NC in power system modeling, Le Boudec and Tomozei (2012) identify strict operation boundaries and the optimal storage size for a battery storage system of an electricity consumer. The objective of the electricity consumer is to satisfy a non-elastic load subject to an external time-varying upper bound on the instantaneous energy consumption.

Wang et al. (2012) use SNC to analyze the impact of the power system's composition on the storage requirements and reliability. They thereby extend the concepts of SNC to power systems by treating them as a simplified queuing problem. For their analysis, the two very specific QoS metrics of the power system, i.e., average Fraction of Time that energy is Not Served (FTNS) and Waste of Power Supply due to improper storage capacity (WPS), are used.

Inspired by these results, Singla et al. (2014) use SNC to optimize the size of a hybrid power system consisting of an energy storage system (ESS) and a diesel backup generator in an unreliable grid to obtain a target carbon footprint.

In the three publications above, the ESS is assumed to have perfect efficiency and only long-term variations of the power supply curves are considered. Ghiassi-Farrokhfal et al. (2014a, 2014b) extend the concepts of a *stochastic power network calculus* (Wang et al. 2012) further by including non-ideal ESS behavior and stochastic process variations on multiple different timescales.

Similar to the worst-case QoS guarantees of NC, operational flexibility can be interpreted as a way to guarantee a desired service, given the expected demand and the technical specification of the energy resources. This analogy gets particularly clear from the work of Weidlich and Zaidi (2019) and Nuytten et al. (2013). In their energy corridor model, possible delays in the operation of electricity-coupled heat generators, such as heat pumps or combined heat and power plants, are estimated based on the system's maximum and minimum operation modes. This method is similar to the concepts of NC to model the minimum delays in computer networks through their respective upper and lower envelopes (Ciucu and Schmitt 2012) as highlighted in Fig. 1.

Since SNC has already been successfully used for modeling power systems (Wang et al. 2012; Ghiassi-Farrokhfal et al. 2014a; Le Boudec and Tomozei 2012), is



Fig. 1 Representation of the analogy of computing minimum delays in computer networks and the possible delays in the operation of an energy device, based on figures from Le Boudec and Thiran (2001); Weidlich and Zaidi (2019)

intrinsically interdisciplinary (Jiang et al. 2016) and shows great similarity to models of operational flexibility (Weidlich and Zaidi 2019; Nuytten et al. 2013), its envelopebased modeling approach is used to develop a generic method to quantify guarantees for the flexibility of a power system in the following section.

# Methodology: SNC concepts for flexibility modeling

In Formulation and notation, fundamental notation is introduced. Then, in Probabilistic power queuing model, the underlying concepts to model the power system as a probabilistic queuing model and the resulting QoS indicators are presented. Subsequently, in Quality of service guarantees, explicit definitions for the worst-case guarantees for power system flexibility calculated from the QoS indicators are derived. Finally, the general approach to model probabilistic bounds on generic QoS indicators in power queuing systems is provided in Modeling methodology.

#### Formulation and notation

The fundamental mathematical notation used in this work to describe the concepts of SNC is summarized from the following publications (Ciucu and Schmitt 2012; Jiang and Liu 2008; Le Boudec and Thiran 2001). In this context, two important functions of the sets F and  $\overline{F}$  are introduced. These functions are the cumulative distribution function (CDF) and the cumulative complementary distribution function (CCDF) of a random variable X denoted by  $F_X(x) = P\{X \le x\}$  and  $\overline{F}_X(x) = P\{X > x\}$ .

Furthermore, for two arbitrary functions f and g the notations  $[f,g]_+ = max\{f,g,0\}$  is used.

The (*min*, +)-convolution, which is characteristic for NC and uses the convention of  $f \otimes g(0) = 0$ , is described by the operator  $\otimes$  in the following way:

**Definition 2** [(*min*, +)-convolution]

$$f \otimes g(x) := \inf_{y \in [0,x]} \left\{ f(y) + g(x - y) \right\}$$
(3)

For any random variables *X* and *Y*, with CCDF  $F_X(x)$  and  $F_Y(x)$  and  $F_X(x) \le \epsilon_X(x)$  and  $F_Y(x) \le \epsilon_Y(x)$ , it is found to hold, whether they are independent or not (Jiang and Liu 2008):

$$P\{X+Y > x\} \le F_X \otimes F_Y(x) \le \epsilon_X \otimes \epsilon_Y(x) \tag{4}$$

In case a function *f* is sub-additive (Van Bemten and Kellerer 2016), which is true if  $f(x + y) \le f(x) + f(y)$  for all *x* and *y*, it is found to hold that  $f \otimes f = f$ . It is further remarked that although a discrete-time model is adopted,  $\inf\{\cdot\}$  and  $\sup\{\cdot\}$  instead of  $\min\{\cdot\}$  and  $\max\{\cdot\}$  operators are used throughout this work.

### Probabilistic power queuing model

In its simplest form, a power grid can be described as a single network node with probabilistic energy demand covered by a controllable or variable energy supply. Excess energy generated is fed into an ESS until fully charged and is discharged whenever there is insufficient energy supply from the source to cover the demand. The energy supplied when the storage is at maximum capacity is said to be curtailed, and energy demand is considered lost when the storage is empty. The flexibility of such a system is its ability to balance a certain energy demand by the additional energy stored in the ESS.

The same system can be described in the computer network analogy by a probabilistic queuing model in which the cumulative energy supply  $E_S(t)$  (in MWh) describes the service process and the cumulative energy demand  $E_A(t)$  (in MWh) the arrival process of the queue (Ardakanian et al. 2012). Time *T* is chosen to be discrete with hourly time steps t. Both energy processes are modeled using a standard SNC Sample-Path-Envelope model following the convention of  $E_A(0) = E_S(0) = 0$ ,  $E_A(s, t) = E_A(t) - E_A(s)$  and  $E_S(s, t) = E_S(t) - E_S(s)$  for any  $0 \le s \le t$ :

**Definition 3** [Stochastic Energy Process] Both an energy demand process  $E_A(t)$  and an energy supply process  $E_S(t)$  are said to have stochastic upper demand  $\alpha_A \in \mathcal{F}$  and supply curve  $\alpha_S \in \mathcal{F}$  with bounding function  $\epsilon_A^u$  or  $\epsilon_S^u \in \overline{\mathcal{F}}$ , denoted by  $E_A \sim \langle \epsilon_A^u, \alpha_A \rangle$  and  $E_S \sim \langle \epsilon_S^u, \alpha_S \rangle$ , if for all  $0 \le s \le t$  and all  $x \ge 0$  there holds:

$$P\left\{\sup_{0\le s\le t}\left\{(E_i(s,t)-\alpha_i(t-s)\right\}>x\right\}\le \epsilon_i^u(x), \quad \text{with } i\in\{A,S\}$$
(5)

and they are said to have stochastic lower demand  $\beta_A \in \mathcal{F}$  and supply curve  $\beta_S \in \mathcal{F}$  with bounding function  $\epsilon_A^l$  or  $\epsilon_S^l \in \overline{\mathcal{F}}$ , denoted by  $E_A \sim \langle \epsilon_A^l, \beta_A \rangle$  and  $E_S \sim \langle \epsilon_S^l, \beta_S \rangle$ , if for all  $0 \le s \le t$  and all  $x \ge 0$  there holds:

$$P\left\{\sup_{0\le s\le t} \{\beta_i(t-s) - E_i(s,t)\} > x\right\} \le \epsilon_i^l(x), \quad \text{with } i \in \{A,S\}$$
(6)

The available charging capacity of the ESS then corresponds to the backlog, i.e., the amount of arrivals that could additionally be served, while the capacity  $C_{ESS}$  of the ESS reflects the buffer size in a conventional queuing system. Both are measured in Watt hours (Wh). Furthermore, the generation curtailment describes a buffer underflow event, while unserved demand describes the buffer overflow process in this analogy (Ardakanian et al. 2012). In the scope of this work, such a queuing system is referred to as a power queuing system. Buffer overflow and buffer underflow events are qualitatively illustrated in Fig. 2. It has to be noted that the power queuing system from computer networking due to its strict QoS constraint.

**Definition 4** [*Quality of Service of Power Systems*] The Quality of Service (QoS) of a power system defines the requirement that the energy demand of end-users is covered at all locations within a user-specific time interval. This time interval is for most end-users instantaneously. In case the coverage cannot be guaranteed, the power system is said to fail its QoS constraint.

This strict QoS constraint means that the departure process of the power queuing system, i.e., the amount of serviced demand, needs to be defined differently from a conventional queuing system. There is no physical interpretation of backlogged energy demand  $E_A(t)$ , as it is either irrecoverably shed or repeatedly shifted until served or shed due to insufficient demand elasticity. This fact requires explicitly considering losses in the power queuing system, e.g., the unserved demand or the generation curtailment, to adequately describe its departure process. Since the departure process itself is unknown, and the service characterization of the power queuing system alone cannot be linked to its arrival process (Wang et al. 2012), SNC is found to be not directly applicable for the QoS analysis of power systems. However, once a departure process is defined that accurately incorporates the QoS constraints of the



**Fig. 2** Representation of a buffer overflow and a buffer underflow event, its meaning, and related terminology in the power queuing system.

power system, the concepts of SNC can be employed to determine guarantees for power system flexibility from a generic set of QoS indicators.

Starting from the assumption of an initially fully charged ESS with capacity  $C_{ESS}$ , the amount of stored energy in the ESS at time t is derived from its standard charging and discharging equations. It can be interpreted as an indicator of the available flexibility potential of the power queuing system at time t. The following non-recursive identity for  $e_{ESS}(t, C_{ESS})$  holds, where the actual storage capacity at time t is represented by the process  $C(t) = C_{ESS} \forall t > 0$  and C(t) = 0 for t = 0 (Wang et al. 2012):

$$e_{ESS}(t, C_{ESS}) = \sup_{0 \le u \le t} \left( \inf_{u \le s \le t} \left( E_S(s, t) - E_A(s, t) + C_{ESS}, E_S(u, t) - E_A(u, t) + C_{ESS} - C(u) \right) \right)$$
(7)

The unserved demand process then characterizes the residual power demand that cannot be met from the ESS at time *t*. It is considered as lost within the power queuing system (Ghiassi-Farrokhfal et al. 2014a). In terms of a conventional power system, the unserved demand describes the load shedding occurring in a power system at time *t*. The following, non-recursive identity for  $E_L(t, C_{ESS})$  is found (Wang et al. 2012):

$$E_{L}(t, C_{ESS}) = \inf_{0 \le u \le t-1} \left( \sup_{u \le s \le t-1} \left( [E_{A}(s, t) - E_{S}(s, t) - C_{ESS}, E_{A}(u, t) - E_{S}(u, t) + C(u) - C_{ESS}]_{+} \right) \right)$$
(8)

From the perspective of power system flexibility,  $E_L(t, C_{ESS})$  describes the energetic flexibility requirements a power system needs to guarantee its QoS. Comparing this finding with the definitions of the backlog and the departure process in SNC (Jiang and Liu 2008), the departure process of the power queuing system is defined as follows:

**Definition 5** (*Departure Process of the Power Queuing System*) The departure process  $E_D(t, C_{ESS})$  of a power queuing system with an ESS of capacity  $C_{ESS}$  describes its actual serviced demand at any point in time t. It is therefore defined as the difference of the desired departure process, given by the arriving energy demand  $E_A(t)$ , and the unserved energy demand  $E_L(t, C_{ESS})$  at each point in time t.

$$E_D(t, C_{ESS}) = E_A(t) - E_L(t, C_{ESS})$$
(9)

It can be noted that Definition 5 accounts for the loss dynamics of the power system in a similar way as the definition of the departure process in SNC for the backlog in computer networks (Jiang and Liu 2008). From this, the additional degree of freedom given by the elasticity of the end-users energy demand in the Definition of the Quality of Service of Power Systems is defined as follows:

$$E_T(t, C_{ESS}) = \inf\{\tau : E_A(t) \le E_D(t + \tau, C_{ESS})\}$$
(10)

The demand elasticity  $E_T(t, C_{ESS})$ , i.e., the ability to flexible shift the demand up to a certain temporal demand deadline, quantifies the time  $\tau$  it takes for the power queuing system until the (desired) requested energy demand  $E_A(t)$  is ultimately served. It describes an indicator of the temporal flexibility requirements of a power system to guarantee its QoS.

Due to directly accounting for the loss dynamics of the power system, the QoS indicators of the power queuing system should be given by the corresponding processes of the QoS indicators of a conventional queuing system. Making use of the general duality principle of queuing systems given by Lemma 3.2 in Raeis et al. (2017), the QoS indicators for the power queuing system are then given by the amount of stored energy in the ESS, the serviced demand and the demand elasticity.

In the following, generic guarantees for the flexibility of power systems under a certain violation probability of its QoS constraints are developed by applying the concepts of SNC to the QoS indicators  $e_{ESS}(t, C_{ESS})$ ,  $E_L(t, C_{ESS})$  and  $E_T(t, C_{ESS})$ .

## **Quality of service guarantees**

Similar to the operational boundaries in Weidlich and Zaidi (2019), Definition 3 defines upper and lower envelopes for the energy demand  $E_A(t)$  and energy supply  $E_S(t)$  process from section Probabilistic power queuing model. Applying these envelopes to Eqs. (7), (9), and (10) allows to derive specific flexibility service guarantees for the power queuing system used in this work. These guarantees are derived under a worst-case condition, e.g., maximum possible power demand  $\alpha_A$  and lowest possible power supply  $\beta_S$ . They are in the following defined as the guaranteed state of charge probability (GSCP), guaranteed power service probability (GPSP), and guaranteed temporal service probability (GTSP). Due to page limitations, the mathematical proof is given in the supplementary information at the following Zenodo archive: http://dx.doi.org/10.5281/ zenodo.8102789. The definitions are general in that no statistical independence assumption is required between energy demand and supply processes.

#### Guaranteed state of charge probability

**Definition 6** [*Guaranteed state of charge probability (GSCP)*] For a stochastic power queuing system, with a probabilistic upper bounded power demand process  $E_A \sim \langle \epsilon_A^u, \alpha_A \rangle$  and probabilistic lower bounded power supply process  $E_S \sim \langle \epsilon_S^l, \beta_S \rangle$ , then for all  $(t \in T : t \ge 0)$  and all  $\gamma \ge 0$ , given a maximum capacity  $C_{ESS}$  of the ESS, the amount of stored  $e_{ESS}(t, C_{ESS})$  in the ESS is bounded by:

$$GSCP(t, C_{ESS}) = P\{e_{ESS}(t, C_{ESS}) \ge \gamma\}$$
  
$$\ge 1 - \epsilon_A^u \otimes \epsilon_S^l (C_{ESS} - \gamma - \sup_{0 \le s \le t} \{\alpha_A(s) - \beta_S(s)\})$$
(11)

The GSCP describes the probability that the amount of stored energy in the ESS exceeds a certain amount of energy  $\gamma$  within the time interval *T*. Thereby,  $\alpha_A(s)$  and  $\beta_S(s)$  are the respective upper and lower envelopes of the energy demand and supply process. The GSCP is then derived as the (min, +)-convolution of the corresponding bounding functions  $\epsilon_A^u \otimes \epsilon_S^l(\cdot)$  using SNC. Regarding power system flexibility, the GSCP describes the guarantee to have at least the amount of energy  $\gamma$  as additional flexibility potential within the power system. A sketch of the mathematical proof is given in the following: *Proof* Fix t > 0, then from Eqs. (7) and (11) it is shown that:  $P\{e_{ESS}(t, C_{ESS}) \ge \gamma\} = P\{\inf_{\substack{0 \le u \le t \ u \le s \le t}} (\sum_{u \le s \le t} (E_A(s, t) - E_S(s, t), E_A(u, t) - E_S(u, t) + C(u))) > C_{ESS} - \gamma\}$ , with its right hand side bounded by  $\sup_{\substack{0 \le s \le t}} (E_S(s, t) - E_A(s, t)) \le \sup_{\substack{0 \le s \le t}} (E_A(s, t) - \alpha_A(t - s)) + \sup_{\substack{0 \le s \le t}} (\beta_S(t - s) - E_S(s, t)) + \sup_{\substack{0 \le s \le t}} (\alpha_A(s) - \beta_S(s)).$ 

By using the Definition of the Stochastic Energy Process for  $E_S(t)$  and  $E_A(t)$ , the rest of the proof then follows from using Eq. (4) and the properties of the (min, +)-convolution. The random processes  $E_S(t)$  and  $E_A(t)$  are not necessarily independent. The full proof is provided in the supplementary information published at the following Zenodo archive: http://dx.doi.org/10.5281/zenodo.8102789.

#### Guaranteed power service probability

**Definition** 7 [*Guaranteed power service probability (GPSP)*] For a stochastic power queuing system, with a probabilistic upper bounded power demand process  $E_A \sim \langle \epsilon_A^u, \alpha_A \rangle$  and a probabilistic lower bounded power supply process  $E_S \sim \langle \epsilon_S^l, \beta_S \rangle$ , the unserved demand  $E_L(t, C_{ESS})$  is bounded by:

$$GPSP(t, C_{ESS}) = P\{E_L(t, C_{ESS}) \le \lambda\}$$
  

$$\ge 1 - \epsilon_A^u \otimes \epsilon_S^l \left(\lambda + C_{ESS} - \sup_{0 \le s \le t} \{\alpha_A(s) - \beta_S(s)\}\right)$$
(12)

The GPSP describes the probability that the unserved demand within a time interval T does not exceed a certain amount of energy  $\lambda$ . It is understood as the required energetic flexibility potential to fulfill a certain QoS guarantee, such as no demand being allowed to be dropped. In the case of power systems, the threshold value  $\lambda$  in Eq. (12) is set to zero to meet its unique QoS constraints given in the Definition of the Quality of Service of Power Systems.  $\lambda$  then describes any event in which the energy demand of endusers is covered, and no load shedding is needed. It is therefore referred to as the load shedding threshold, as it sets the energy that constitutes the demand-supply imbalance, above which a QoS violation is said to occur.

#### Guaranteed temporal service probability

**Definition 8** [*Guaranteed temporal service probability (GTSP)*] For a stochastic power queuing system, with a probabilistic upper bounded power demand process  $E_A \sim \langle \epsilon_A^u, \alpha_A \rangle$  and a probabilistic lower bounded power supply process  $E_S \sim \langle \epsilon_S^l, \beta_S \rangle$ , the demand elasticity  $E_T(t, C_{ESS})$  is bounded by:

$$GTSP(t, C_{ESS}) = P\{E_T(t, C_{ESS}) \le \tau\}$$
  
$$\ge 1 - \epsilon_A^u \otimes \epsilon_S^l \left( C - \sup_{0 \le s \le t} \{ \alpha_S(s) - \beta_S(s + \tau) \} \right)$$
(13)

The GTSP describes the probability that the demand elasticity  $E_T(t)$  within a time interval T is shorter than a certain amount of time  $\tau$ . It is understood as the required temporal flexibility potential to fulfill a certain QoS guarantee, such as each demand being served within two hours. In the case of power systems, the threshold value  $\tau$  in Eq. (13)

reflects the user-specific time interval to cover its demand as indicated in the Definition of the Quality of Service of Power Systems.  $\tau$  is therefore referred to as the demand elasticity and sets the time for the demand-supply balance, after which a QoS violation is said to occur.

It is interesting to notice that for an instantaneous demand-supply balance, e.g., no demand elasticity  $\tau = 0$  and no load shedding  $\gamma = 0$ , Eq. (13) and Eq. (12) are the same. From this, the GTSP is identified to be also an indicator of the duration of demand-supply imbalance in power systems, while the GPSP accounts for its frequency. The following sections present a generic modeling method to calculate the probabilistic envelopes from section Probabilistic power queuing model and the guarantees for power system flexibility presented in section of Quality of service guarantees.

## Modeling methodology

In this work, the energy demand  $E_A(t)$  and the energy supply  $E_S(t)$  are modeled as given in Definition Stochastic Energy Process using a standard SNC SNC Sample-Path-Envelope approach. The envelope functions of both energy demand  $E_A(t)$  and the energy supply  $E_S(t)$  are chosen to be affine functions of the form  $\alpha_i = [(\rho + \delta) \cdot t + \sigma]_+$  and  $\beta_i = [(\rho - \delta) \cdot t - \sigma]_+$ , with  $i \in \{A, S\}$ .

Thereby,  $\rho$  describes the rate parameter of the underlying energy process and is set to its long-term average mean, e.g.,  $\rho_i = \sum_{t=1}^T \frac{E_i(t)}{T}$ . Meanwhile,  $\delta$  and  $\sigma$  are tuning parameters of the envelopes, which resemble the rate variation and burstiness of the underlying random process (Ciucu and Schmitt 2012).

The corresponding bounding functions of both stochastic energy processes are chosen to be simple exponential decays of the form  $\epsilon_i(x) = p \cdot \exp\{-\kappa \cdot x\}$ . The initial amplitude p and the decay rate  $\kappa$  of the bounding function  $\epsilon_i(x)$  are derived from fitting the bounding function to the CCDF of the envelope model given in Definition 3. The general procedure for calculating the bounding functions for the power demand and supply process  $E_i^j(t)$  given a set of measurement traces J then takes the following steps:

- Envelope Model: Choose a specific set of envelope function α<sup>j</sup><sub>i</sub>(t) and β<sup>j</sup><sub>i</sub>(t) for the energy demand and supply process E<sup>j</sup><sub>i</sub>(t), with j ∈ J. In this work, the same affine functions of the form [(ρ ± δ) · t ± σ]<sub>+</sub> are chosen for each measurement trace.
- 2. Envelope Violations: Calculate the maximum deviations, e.g., the left-hand-side term (LHS) in Definition 3,  $LHS_i^{j,t}$ , of the chosen envelope model  $\alpha_i^j(t)$  or  $\beta_i^j(t)$  and the measurement traces  $E_i^j(t)$  for all  $t \leq 0$ :

$$LHS_i^{j,t,u} = \sup_{0 \le s \le t} \{E_i^j(t)(s,t) - \alpha_i(t-s)\}$$
$$LHS_i^{j,t,l} = \sup_{0 \le s \le t} \{\beta_i(t-s) - E_i^j(t)(s,t)\},$$
with  $i \in \{A, S\}$  and  $j \in J$ 

It should be noted that all elements of the LHS must be non-negative. Therefore, all negative elements of LHS are set to zero.

- 3. Bounding Function: In general, the CCDF of any distribution that fits the tail of the LHS can be chosen as a sufficiently good approximation of the respective bounding function  $\epsilon_i^u(x)$  and  $\epsilon_i^l(x)$  (Ciucu and Schmitt 2012). This work assumes exponential tail distributions, and simple exponential decays of the form  $p \cdot \exp\{-\kappa \cdot x\}$  are used as bounding functions.
- 4. Parametrization: The decay rate  $\kappa$  of the bounding functions are obtained by curve fitting the non-zero distribution (*LHS* > 0) of the LHS over the free parameter x in Definition 3. The non-zero distribution of LHS is chosen to avoid bias of the zero elements in the fitted distribution. Furthermore, the initial amplitude p of the bounding functions is set to the ratio of non-zero elements and the total number of elements in the LHS (Ghiassi-Farrokhfal et al. 2014a).

For a given set of measurement traces  $E_A^j(t)$  and  $E_S^j(t)$ , there are infinitely many combinations of envelope and bounding functions to choose from. However, not all combinations yield useful results. Balancing the tuning of the envelope functions  $\alpha_i(t)$  and  $\beta_i(t)$ with the resulting bounding functions  $\epsilon_i^u(x)$  and  $\epsilon_i^l(x)$  involves a certain trade-off. Tightening the envelope function increases the bounding function and vice versa. Ideally, the optimal combination should consider both high fluctuations in the energy demand and energy supply processes (Wang et al. 2012).

Recall that each flexibility service guarantee is constrained by the (min, +)-convolution between the two processes, which is not only computationally expensive but even amplifies the trade-off between envelope and bounding functions further since both energy demand and energy supply must be optimized simultaneously (Wang et al. 2012). When exponential bounding functions are employed, Lemma 3 from Ciucu et al. (2006) drastically decreases the computation time of the (min, +)-convolution. The optimal guarantees for power system flexibility, as defined in the section Quality of service guarantees, are then the result of solving a joint optimization problem over the free tuning parameters  $\sigma_i$  and  $\delta_i$  for both energy processes  $E_A(t)$  and  $E_S(t)$ .

In practical applications, a feasibility space for the tuning parameters  $\sigma_i$  and  $\delta_i$  that adequately captures a broad range of fluctuations is chosen for both energy processes. Note that the feasibility space grows with  $O(n^2)$ , which is why knowledge of the modeled process can reduce computation time drastically. Since the optimization problem is twofold, all combinations of the envelope and bounding functions within the feasibility space are first calculated. Then, the second step optimizes the flexibility service guarantees for different system compositions. The optimal flexibility service guarantees are given by the maximum probabilities among all feasible envelope and bounding function combinations. The algorithm developed for this work is published in the following Zenodo archive: http://dx.doi.org/10.5281/zenodo.8102789. In the following, the modeling approach is validated in a case study on the reliability of a solar-powered car charging station.

## Case study: reliability of solar-powered car charging

The case study investigates and validates the capabilities of computer networkingbased guarantees for power system flexibility to quantify the reliability of a solarpowered car charging station for a small research institute in Germany.

# Structure and data

The power system under consideration consists of ten car charging stations in parallel, a lossless ESS with capacity  $C_{ESS}$ , and renewable power generation units in the form of solar panels. The car charging profiles are based on the German mobility survey (Nobis and Kuhnimhof 2018) and are obtained from emobpy (Gaete-Morales et al. 2021). The data set consists of 200 car charging profiles in 15-min resolution distributed across four car types, from which 97 are commuting full-time workers, and 27 are commuting parttime workers. The per unit solar generation profiles are obtained in 15-min resolution from Pfenninger and Staffell (2016) and the associated web platform (www.renewables. ninja) for the model years 2011 to 2020. The hourly variations and the average cumulative energy of both data sets are depicted in Fig. 3 for representative weeks in summer ( $a_1$  to  $c_1$ ) and winter ( $a_2$  to  $c_2$ ).

The solar generation profiles exhibit a typical daily pattern, with significantly more generation in summer than in winter due to meteorological factors. In contrast, the workplace car charging profiles exhibit much stronger hourly variations and a slightly increased cumulative energy demand in winter. This difference is most likely due to decreasing efficiency of electric vehicles with colder temperatures (Jaguemont et al. 2016).



**Fig. 3** Hourly variations and cumulative sum of the mean profiles of the per unit solar generation (blue) and car charging demand (orange) for representative weeks in summer and winter, with the standard deviation included as a colored error band

## Modeling approach

The guarantees for power system flexibility are calculated according to the section Modeling methodology. The measurement traces for the energy demand process  $E_A^j(t)$  are given by one hundred random combinations of ten workplace car charging profiles from the emobpy dataset (Gaete-Morales et al. 2021) for eight representative weeks in summer and winter. The split between full-time and part-time workers was set to eight over two. The total power supply was modeled using ten years (2011 to 2020) of per unit solar generation profiles from Pfenninger and Staffell (2016) for the same representative weeks in summer and winter. The solar generation units were assumed to be homogeneous. This assumption allows direct scaling of the per-unit solar profiles with the installed capacity of interest due to the sub-additivity of the corresponding bounding functions. If heterogeneous power generation units are used, an additional aggregation step is required to transform the multiple supply curves into a single one. For further details, the interested reader is referred to Wang et al. (2012).

As sample path envelopes affine functions of the  $[(\rho \pm \delta) \cdot t \pm \sigma]_+$  are used with the rate parameter  $\rho$  set to the long-term mean rate of the used measurement traces. The tuning parameters for each envelope  $\sigma$  and  $\delta$  are varied in a total of 300 combinations, showing robust results against wider or smaller step sizes once a minimum number of combinations was used. The bounding functions are given by a simple exponential decay of the form  $p \cdot \exp\{-\kappa \cdot x\}$ . Then based on the section Modeling methodology, all feasible combinations of the envelope and bounding functions for the power demand and supply processes are calculated. A selection of stochastic envelopes in comparison to the average cumulative generation and charging profiles are depicted in Fig. 3. Finally, the optimal guarantees for power system flexibility are calculated for different ESS capacities  $C_{ESS}$  and threshold values  $\gamma$ ,  $\lambda$ , and  $\tau$ . The detailed parameters used for the optimization are accessible in the source code published at the following Zenodo archive: http://dx.doi.org/10.5281/zenodo.8102789.

In order to validate the results of the computer networking-based modeling approach, it is compared with a MILP using the open-source framework PyPSA (Brown et al. 2018). The objective function of the problem is an operational cost minimization (Brown et al. 2018) with perfect foresight and certain demand elasticity as an additional constraint. Since all marginal generation costs are set to zero, the problem optimizes the dispatch of the solar generation and the storage unit to fulfill the demand. In order to ensure the feasibility of the optimization, backup generation is available at high marginal costs. The input data used in the MILP approach is the same as for the SNC approach.

In contrast, to the SNC approach, which solves the underlying statistics of the problem (1200 weeks) in advance, the MILP optimization requires solving for multiple random samples to capture the statistics of the problem afterward. Therefore one hundred random model runs (100 weeks) were evaluated for the same parameter combinations used in the SNC approach. This method comes at relatively high computational costs, with the SNC approach being roughly 100 times faster than the MILP approach by a highly increased number of random samples (1200 weeks vs. 100 weeks).

In both models, the capacity of the ESS  $C_{ESS}$  is assumed to be time-invariant and initially fully charged ( $e_{ESS}(0) = C_{ESS}$ ). Furthermore, a cyclic state-of-charge constraint of the form  $e_{ESS}(t_{end}) \ge e_{ESS}(t_{init})$  is applied to have no free energy in the system, e.g., physical feasibility. This constraint could lead to different results than published in previous work (Wang et al. 2012; Singla et al. 2014; Ghiassi-Farrokhfal et al. 2014a). A trivial example would be a power system with an energy demand  $E_A(t)$  that is always larger than the average renewable generation  $E_S(t)$ . In such a system, even an ESS of infinite size could not guarantee the system's power supply below a certain violation probability  $\epsilon$ .

## Numerical results

The following presents the resulting guarantees for power system flexibility obtained from computer networking concepts. First, in section Comparison of flexibility service guarantees, the flexibility service guarantees obtained from the SNC and the MILP modeling approach are compared for an increasing amount of system flexibility. Second, section Influence of increasing solar capacity investigates the impact of increasing solar generation on the flexibility service guarantees. Finally, section Influence of increasing solar capacity analyzes the influence of varying seasonality.



Flexibility Guarantees for a Solar Capacity of 2.53 kW in Summer

Fig. 4 Comparison of the flexibility service guarantees obtained from the stochastic network calculus and mixed-integer linear programming modeling approach for different energy storage system capacities and a solar capacity of 2.53 kW in the summer season

#### Comparison of flexibility service guarantees

In Fig. 4, the guarantees for power system flexibility are depicted for different threshold values  $\gamma$ ,  $\lambda$ , and  $\tau$  and an increased capacity of the ESS. They represent the worstcase guarantees for the time interval of one week in the summer season. For validation the results from both the SNC ( $a_1$ ,  $b_1$ ,  $c_1$ ) and the MILP modeling approach ( $a_2$ ,  $b_2$ ,  $c_2$ ), as well their difference (SNC - MILP; ( $a_3$ ,  $b_3$ ,  $c_3$ )), are compared. The installed capacity of the solar generation unit was dimensioned to match the average daily car charging demand to a size of 2.53 kW for a representative week in the summer season. It is observed that all three QoS indicators exhibit the same trend: With increasing system flexibility, in the form of an increased ESS capacity  $C_{ESS}$  or demand elasticity  $\tau$ of the car charging requests, the flexibility service guarantees are increased.

Figure 4 (*a*<sub>1</sub>) and (*a*<sub>2</sub>) illustrate the GSCP for various states of charge (SOC) of the ESS. The SOC is defined as the ratio between the stored energy in the ESS at time *t* and the ESS capacity  $C_{ESS}$  to  $SOC(t, C_{ESS}) = \frac{e_{ESS}(t, C_{ESS})}{C_{ESS}}$ . The GSCP represents the likelihood that a power system will exhibit additional flexibility potential  $\gamma$  equal to or greater than the stored energy of the ESS with capacity  $C_{ESS}$ . Both the SNC (*a*<sub>1</sub>) and the MILP (*a*<sub>2</sub>) approach yield comparable results. For example, for an ESS with capacity greater than 25 kWh, both approaches show low flexibility service guarantees (< 25 %) for SOC values below 60% of the ESS capacity.

Furthermore, Fig. 4 ( $b_1$ ) and ( $b_2$ ) display the GPSP for a load shedding threshold of  $\lambda = 0$ . The GPSP indicates the likelihood that a power system, with an energetic flexibility potential given by the capacity  $C_{ESS}$  of an ESS, meets the QoS constraint of power systems specified in Definition 4. The maximum flexibility service guarantees are approximately 80% for both the SNC approach ( $b_1$ ) and the MILP approach ( $b_2$ ), with a saturation of the GPSP occurring for ESS capacities of 27.3 kWh and 36.4 kWh, respectively.

Finally, Fig. 4 ( $c_1$ ) and ( $c_2$ ) present the GTSP for increasing demand elasticity  $\tau$ . The GTSP indicates the likelihood that a power system, with a temporal flexibility potential determined by the elasticity of the demand  $\tau$ , meets the QoS constraint of power systems specified in Definition 4. For both modeling approaches, the GTSP increases along the  $C_{ESS}$  and the  $\tau$  axis. The temporal flexibility of the power demand is dependent on the ESS capacity, with smaller capacities having a larger influence.

This finding suggests that for larger ESS capacities, any further increase in flexibility service guarantees can only be achieved by increasing the temporal flexibility of the system since the ESS's flexibility is already optimally utilized. Additionally, the transition from low (red) to high (blue) flexibility service guarantees is observed to differ between the SNC and MILP approaches. The SNC approach ( $c_1$ ) exhibits a strictly linear increase in flexibility service guarantees, while the MILP approach ( $c_2$ ) shows a stair-like increase at demand elasticity  $\tau$  values around 4 and 20 hours.

In general, the SNC approach overestimates the flexibility service guarantees in all three cases, which is demonstrated in Fig. 4 ( $a_3$ ) to ( $c_3$ ). It shows the differences between the SNC and the MILP approach for each guarantee. The differences in results can be attributed to the advantages of the MILP approach, which has perfect foresight and optimally utilizes the flexibility of the ESS and the elasticity of car charging requests. In

addition, the SNC approach employs a simplified envelope model that does not account for the inter-hourly variability of solar generation and car charging requests.

Furthermore, the MILP approach is purely deterministic and has strict decision criteria, meaning that the QoS constraint of power systems specified in Definition 4 can either be fulfilled or not. In contrast, the SNC approach is probabilistic, allowing for the incorporation of low violations of the QoS constraint of the power system, which can lead to better guarantees, which is observed in the results of Fig. 4. Nonetheless, the MILP approach confirms the results of the SNC modeling approach, which provides confidence that the chosen SNC approach is suitable for modeling power system flexibility. In the following, the influence of an increased installed solar capacity and different seasonality is only discussed for the SNC approach.

#### Influence of increasing solar capacity

In Fig. 5a, the GTSP is depicted for an increased capacity of the installed solar generation unit of 2.53 kW. The resulting flexibility service guarantees are significantly higher, which can be attributed to the better utilization of the existing system flexibility. The dependencies between the installed generation capacity and the required ESS capacity  $C_{ESS}$  and the elasticity  $\tau$  of the power demand are depicted in Fig. 5b and c. Small increases in generation capacity substantially impact the flexibility service guarantees of the power system when sufficient flexibility is available. Figure 5d provides additional quantification of the optimal system composition by showing the required energetic and temporal flexibility potential to achieve a QoS guarantee of at least 90 % for various solar generation capacities. This potential is measured in terms of the ESS capacity  $C_{ESS}$  and the power demand elasticity  $\tau$ . Although the base system's power generation capacity is set at 2.53 kW and cannot meet the desired QoS guarantee regardless of the



GTSP for Increasing Solar Capacity in Summer

**Fig. 5** Graphical representation of the guaranteed temporal service probability for an increased solar capacity of 3.79 kW, as well as the dependencies between the installed generation capacity and the required energy storage system capacity  $C_{ESS}$  and the demand elasticity  $\tau$ 



#### GPSP for a Solar Capacity of 6.32 kW

**Fig. 6** Graphical representation of the guaranteed power service probability for a fixed solar capacity of 6.32 kW for representative weeks in summer and winter.

additional ESS capacity  $C_{ESS}$  and demand elasticity  $\tau$ , a mere 20% increase in solar generation capacity is enough to achieve the desired QoS guarantee. Further power supply increase can lead to system solutions requiring low amounts of ESS capacity  $C_{ESS}$  and no demand elasticity  $\tau$ .

## Meteorological impact

Finally, the impact of different meteorological conditions on the flexibility service guarantees of power system flexibility is analyzed. Figure 6 shows the GPSP for a power system with an installed solar generation capacity of 6.32 kW for representative weeks in summer and winter. As expected, the energetic flexibility potential to guarantee a certain QoS level given by the GPSP is much higher in summer than in winter. For instance, the probability of a power system to meet its QoS constraint specified in Definition 4 given an ESS with a capacity of 15 kWh as energetic flexibility potential is above 95 % in summer, but only roughly around 50 % in winter. This result can be attributed to two factors. First, during summer, there is a substantial increase in solar power generation due to favorable meteorological conditions. Second, during winter, there is an increase in energy demand resulting from the lower efficiency of electric vehicles in colder temperatures.

# Discussion

The QoS guarantees derived in this work using SNC concepts offer a statistical quantification of power system flexibility under worst-case conditions. The main advantage of this approach is its ability to analytically derive flexibility service guarantees across multiple time intervals rather than focusing on individual time points. By incorporating underlying statistics in advance, the computational time is significantly reduced compared to conventional simulation methods since these require retrospective consideration of statistics. Moreover, the results for a specific model configuration can be applied to other systems with similar dynamics.

However, it is essential to acknowledge a notable limitation of the current modeling approach. The underlying Sample-Path-Envelope model accounts for the history of energy demand and supply processes and does not directly capture the temporal dynamics of flexibility potentials and requirements. The flexibility service guarantees derived in this work represent worst-case scenarios over a week-long interval rather than at specific time points. One potential solution to address this limitation is to modify the envelope model to incorporate the power system's temporal dynamics directly. An alternative envelope model worth exploring are traffic-amount-centric (t.a.c.) envelopes (Ciucu and Schmitt 2012; Jiang and Liu 2008). Unlike the sample path envelopes used in this work, t.a.c. envelopes do not include worst-case situations of previous time intervals. However, it should be noted that t.a.c. envelopes currently do not provide the desired QoS indicators of computer networking without imposing additional constraints.

## Conclusion

This work explores the use of computer networking concepts to quantify guarantees for power system flexibility. The motivation for this research arises from the similarities between Quality of Service indicators in computer networking and the operational flexibility required to ensure desired Quality of Service levels in the power system. Due to its unique envelope-based modeling approach, the framework of stochastic network calculus is employed to generate and aggregate stochastic envelopes for random power demand and supply processes at various time scales.

Using the concepts of stochastic network calculus, guarantees for power system flexibility are developed and validated through a case study on the reliability of a solar-powered car charging station. Three guarantees are established: guaranteed state of charge probability, the guaranteed power service probability, and the guaranteed temporal service probability. These guarantees assess the presence and sufficiency of specific flexibility potentials to ensure the desired QoS level and are represented either by the capacity of an energy storage system or the demand elasticity of the power demand.

Results indicate that the flexibility service guarantees derived from the stochastic network calculus approach align well with common approaches in mixed-integer linear programming modeling while significantly reducing computation times and resource requirements. Its main advantage is the analytical derivation of Quality of Service indicators for power system flexibility across multiple time scales.

Future work could involve analyzing the impact of different envelope models and expanding the analysis to include multiple network nodes, encompassing transmission and market constraints considerations.

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#### Author contributions

TF: Conceptualization, investigation, methodology, visualization, writing—original draft; ML: Conceptualization, methodology, writing—review and editing; HM: Supervision, writing—review and editing, funding acquisition; AW: Conceptualization, supervision, writing—review and editing, funding acquisition.

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#### Availability of data and materials

The source code of the optimization algorithm, along with its description of the algorithm in pseudo-code and mathematical proof, is available at the following Zenodo archive: http://dx.doi.org/10.5281/zenodo.8102789.

## Declarations

#### **Competing interests**

The authors declare that they have no competing interests.

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